MATURA
maj 2017r.
poziom rozszerzony

PRZYKŁADOWE ROZWIĄZANIA

Zad. 1 <1 pkt.>

$$\left(\sqrt{2-\sqrt{3}} - \sqrt{2+\sqrt{3}}\right)^2 =$$

$$= 2-\sqrt{3} + 2+\sqrt{3} - 2\sqrt{(2-\sqrt{3})(2+\sqrt{3})} =$$

$$= 4 - 2\sqrt{4-3} = 4-2 = \underline{\underline{2}}$$

A

Zad. 2 <1 pkt.>

$$a_n = \frac{(n^2-10n)(2-3n)}{2n^3+n^2+3}$$

$$\lim_{n\to\infty} a_n = ?$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{-3n^3+2n^2+15n^2-20n}{2n^3+n^2+3} = -\frac{3}{2}$$

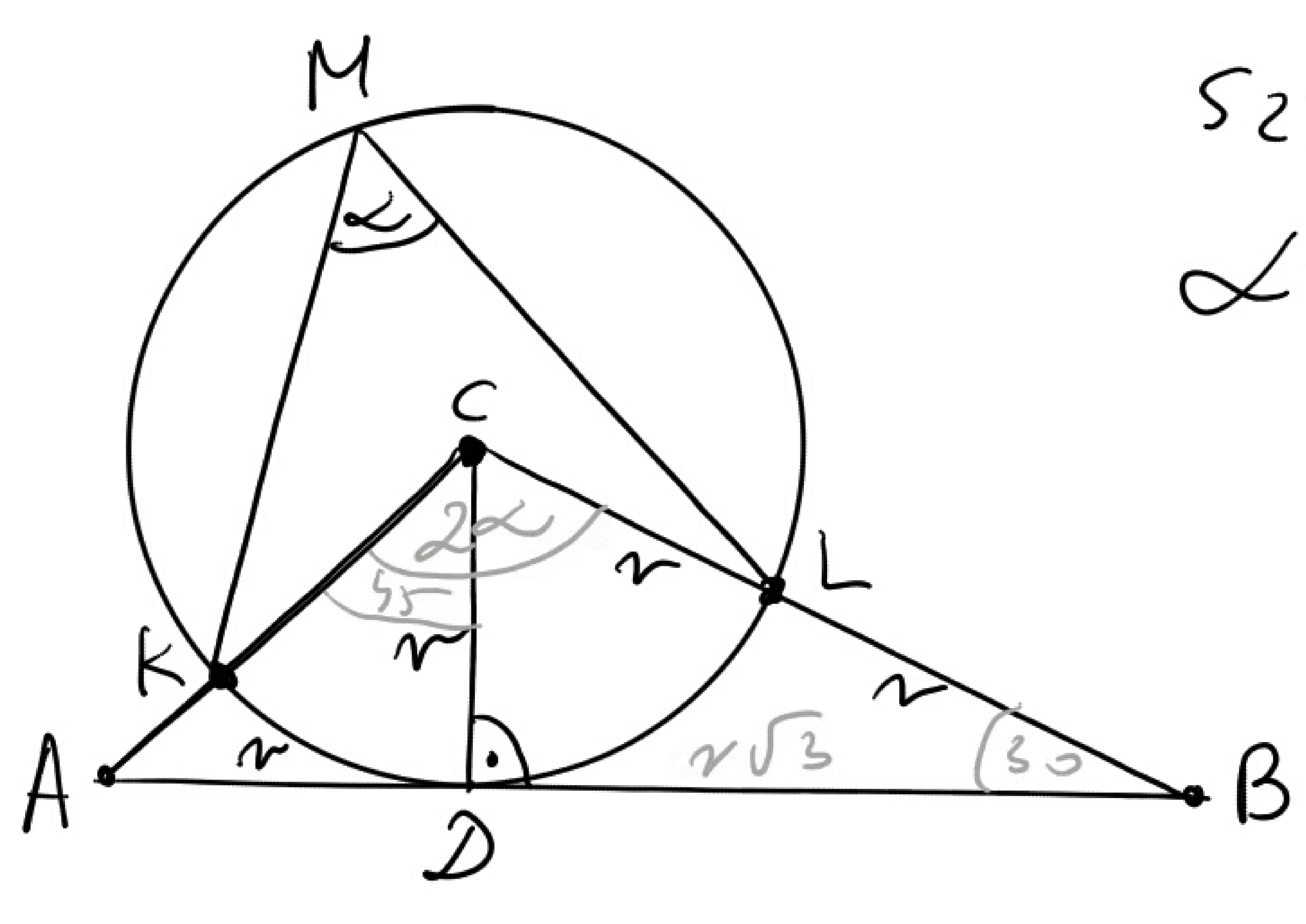
D

Zad. 3 <1 p.>

D: $|AD| = |DC| = r > 0$

$|BC| = 2r$

Sz: $\alpha = ?$



① $\overset{\frown}{KL}$:

$|\sphericalangle KCL| = 2\cdot\alpha$

② $\triangle ADC$: $45°, 45°, 90° \longrightarrow |\sphericalangle ACD| = 45°$

$\triangle DBC$: $30°; 60°; 90° \longrightarrow |\sphericalangle DCB| = 60°$

③ $2\alpha = 45° + 60° = 105°$

$\alpha = 52{,}5°$

C

Zad. 4 <1 pkt>

$A = (x_A; y_A) = ?$ | $B = (-4; 7)$, $\vec{u} = [-3; 5]$

① $\vec{AB} = -3\vec{u}$

①

$[-4 - x_A; 7 - y_A] = -3 \cdot [-3; 5]$

$[-4 - x_A; 7 - y_A] = [9; -15] \Rightarrow \begin{cases} -4 - x_A = 9 \\ 7 - y_A = -15 \end{cases}$

B $\quad A = (-13; 22) \Leftarrow \begin{cases} x_A = -13 \\ y_A = 22 \end{cases}$

Zad. 5 <2 pkt.>

$W(x) = x^3 - 2x^2 + ax + \frac{3}{4}$ | $a = ?$

$W(2) =$

$8 - 8 + 2a + \frac{3}{4} = 1$

$2a = \frac{1}{4} \quad | : 2$

$a = \frac{1}{8} = 0{,}125$

Zad. 6 <3 pkt.>

$f(x) = \frac{x-1}{x^2+1}$

$P = (1; 0) = (x_0; y_0)$ | $s: y = f'(x_0)(x - x_0) + f(x_0) = ?$

① $f'(x) = \frac{1 \cdot (x^2+1) - (x-1) \cdot 2x}{(x^2+1)^2} = \frac{x^2 + 1 - 2x^2 + 2x}{(x^2+1)^2}$

$f'(x) = \frac{-x^2 + 2x + 1}{(x^2+1)^2} \Rightarrow f'(1) = \frac{-1+2+1}{(1+1)^2} = \frac{2}{4} = \frac{1}{2}$

②

$s: y = \frac{1}{2}(x - 1) + 0 \Rightarrow s: y = \frac{1}{2}x - \frac{1}{2}$

Zad. 7 (3 pkt.)

Zał.: $x \neq y \quad \wedge \quad x, y \in \mathbb{R}$

Teza: $x^2y^2 + 2x^2 + 2y^2 - 8xy + 4 > 0$

I metoda:

Dowód: $x^2y^2 + 2(x-y)^2 - 4xy + 4 > 0$

$$\underbrace{\underbrace{2(x-y)^2}_{>0} + \underbrace{(xy-2)^2}_{\geq 0}}_{>0} > 0$$

$\bigwedge_{\substack{x,y\in\mathbb{R}\\ x\neq y}}$

cnd.

II metoda:

Dowód: $\underbrace{(y^2+2)x^2 - 8yx + 2y^2+4}_{f(x)} > 0$

$a = y^2+2 \; ; \; b = -8y \; ; \; c = 2y^2+4 = 2(y^2+2)$

$f(x) > 0 \implies \Delta_x < 0$

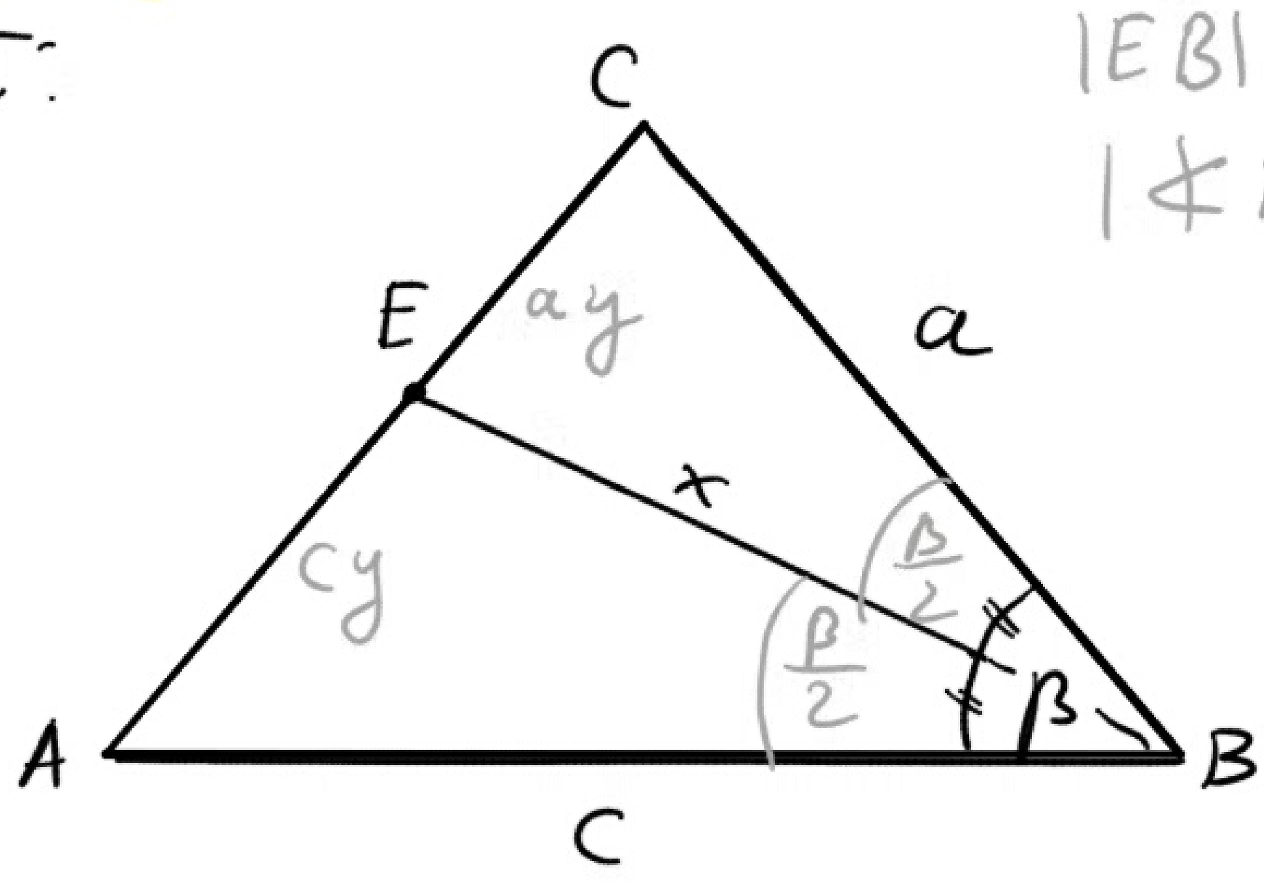
$$(-8y)^2 - 4\cdot(y^2+2)\cdot 2(y^2+2) < 0$$
$$16y^2 - 8(y^2+2)^2 < 0 \qquad /:8$$
$$2y^2 - (y^2+2)^2 < 0$$
$$2y^2 - y^4 - 4y^2 - 4 < 0$$
$$-y^4 - 2y^2 - 4 < 0 \qquad /:(-1)$$
$$y^4 + 2y^2 + 4 > 0$$
$$(y^2+2)^2 > 0$$

$\bigwedge_{x,y\in\mathbb{R}}$

cnd.

Zad. 8 <3pkt.>

Zał:



$|EB| = x > 0$
$|\sphericalangle ABC| = \beta$

Teza:

$$x = \frac{2ac \cdot \cos\frac{\beta}{2}}{a+c}$$

I metoda:

D: ① $|AE| = c \cdot y \ \wedge \ |EC| = ay \Rightarrow$ z tw. o dwusiecznej

$$\cos\beta = \cos\left(2 \cdot \frac{\beta}{2}\right) = 2\cos^2\frac{\beta}{2} - 1$$

② Tw. cosin. dla $\triangle ABC$, $\triangle ABD$ i $\triangle EBC$:

$$\begin{cases} \boxed{1}\ (ay+cy)^2 = a^2+c^2-2ac\cos\beta \\ \boxed{2}\ (cy)^2 = c^2+x^2-2cx\cos\frac{\beta}{2} \\ \boxed{3}\ (ay)^2 = a^2+x^2-2ax\cos\frac{\beta}{2} \end{cases} \wedge \quad \cos\beta = 2\cos^2\frac{\beta}{2} - 1$$

$\boxed{1}$ $y^2(a+c)^2 = a^2+c^2-2ac\left(2\cos^2\frac{\beta}{2}-1\right)$

$y^2(a+c)^2 = a^2+c^2+2ac-4ac\cos^2\frac{\beta}{2}$

$y^2(a+c)^2 = (a+c)^2-4ac\cos^2\frac{\beta}{2} \qquad |:(a+c)^2 \neq 0$

$\boxed{1}$ $y^2 = 1 - \frac{4ac\cdot\cos^2\frac{\beta}{2}}{(a+c)^2} = \frac{(a+c)^2-4ac\cos^2\frac{\beta}{2}}{(a+c)^2}$

$\boxed{3-2}$ $y^2(a^2-c^2) = a^2-c^2-2x(a-c)\cdot\cos\frac{\beta}{2} \qquad |:(a-c)\neq 0$

$y^2(a+c) = (a+c) - 2x\cos\frac{\beta}{2} \qquad \wedge \quad \boxed{1}$

$\frac{(a+c)^2-4ac\cos^2\frac{\beta}{2}}{a+c} = (a+c) - 2x\cos\frac{\beta}{2} \qquad |\cdot(a+c)$

$\cancel{(a+c)^2} - 4ac\cos^2\frac{\beta}{2} = \cancel{(a+c)^2} - 2(a+c)x\cos\frac{\beta}{2} \qquad |:\left[-2\cos\frac{\beta}{2}\right]\neq 0$

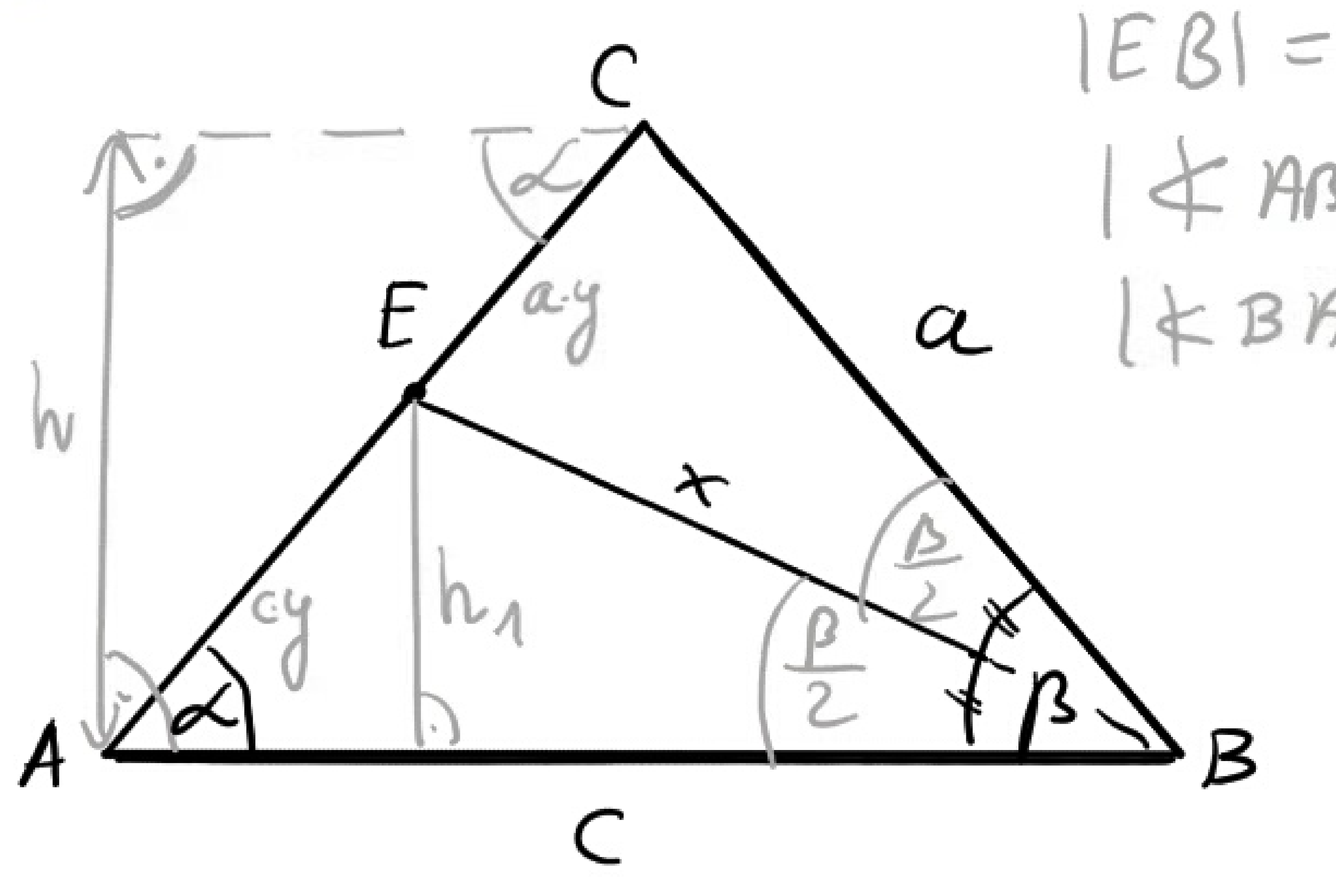
$2ac\cos\frac{\beta}{2} = (a+c)x \qquad |:(a+c)\neq 0$

$$x = \frac{2ac\cos\frac{\beta}{2}}{a+c}$$

cnd.

Zad. 8 (3 pkt.)

Zał:



$|EB| = x > 0$
$|\sphericalangle ABC| = \beta$
$|\sphericalangle BAC| = \alpha$

Teza:

$$x = \frac{2ac \cdot \cos\frac{\beta}{2}}{a+c}$$

II metoda

D: ① $|AE| = c \cdot y \;\wedge\; |EC| = ay \Rightarrow$ z tw. o dwusiecznej

②

$$P_{ABC} = \frac{1}{2} \cdot c \cdot h = \frac{1}{2} ac \sin\beta \Rightarrow \underbrace{h} = a \cdot \sin\beta = \underbrace{2a \sin\frac{\beta}{2} \cdot \cos\frac{\beta}{2}}$$

③

$$\sin\alpha = \frac{h}{ay+cy} = \frac{h_1}{cy} \quad / \cdot y$$

$$\frac{h}{a+c} = \frac{h_1}{c} \Rightarrow \underbrace{h_1} = \frac{c \cdot h}{a+c} = \frac{2ac \sin\frac{\beta}{2} \cos\frac{\beta}{2}}{a+c}$$

④

$$P_{ABE} = \frac{1}{2} c \cdot h_1 = \frac{1}{2} cx \sin\frac{\beta}{2} \quad / \cdot \frac{2}{c} > 0$$

$$h_1 = x \cdot \sin\frac{\beta}{2} \Rightarrow \underbrace{x = \frac{h_1}{\sin\frac{\beta}{2}}}$$

$$x = \frac{2ac \cancel{\sin\frac{\beta}{2}} \cos\frac{\beta}{2}}{(a+c) \cdot \cancel{\sin\frac{\beta}{2}}} = \frac{2a \cdot c \cdot \cos\frac{\beta}{2}}{a+c}$$

cnd.

Zad. 9 (4pkt.)

D:

$|AB| = |BC| = |AC| = |AW| = 6 = a$

$H = |OW| > 0$; $H_1 = |PW| > 0$

$h = |DW| > 0$

$R = |SK| = |SO|$

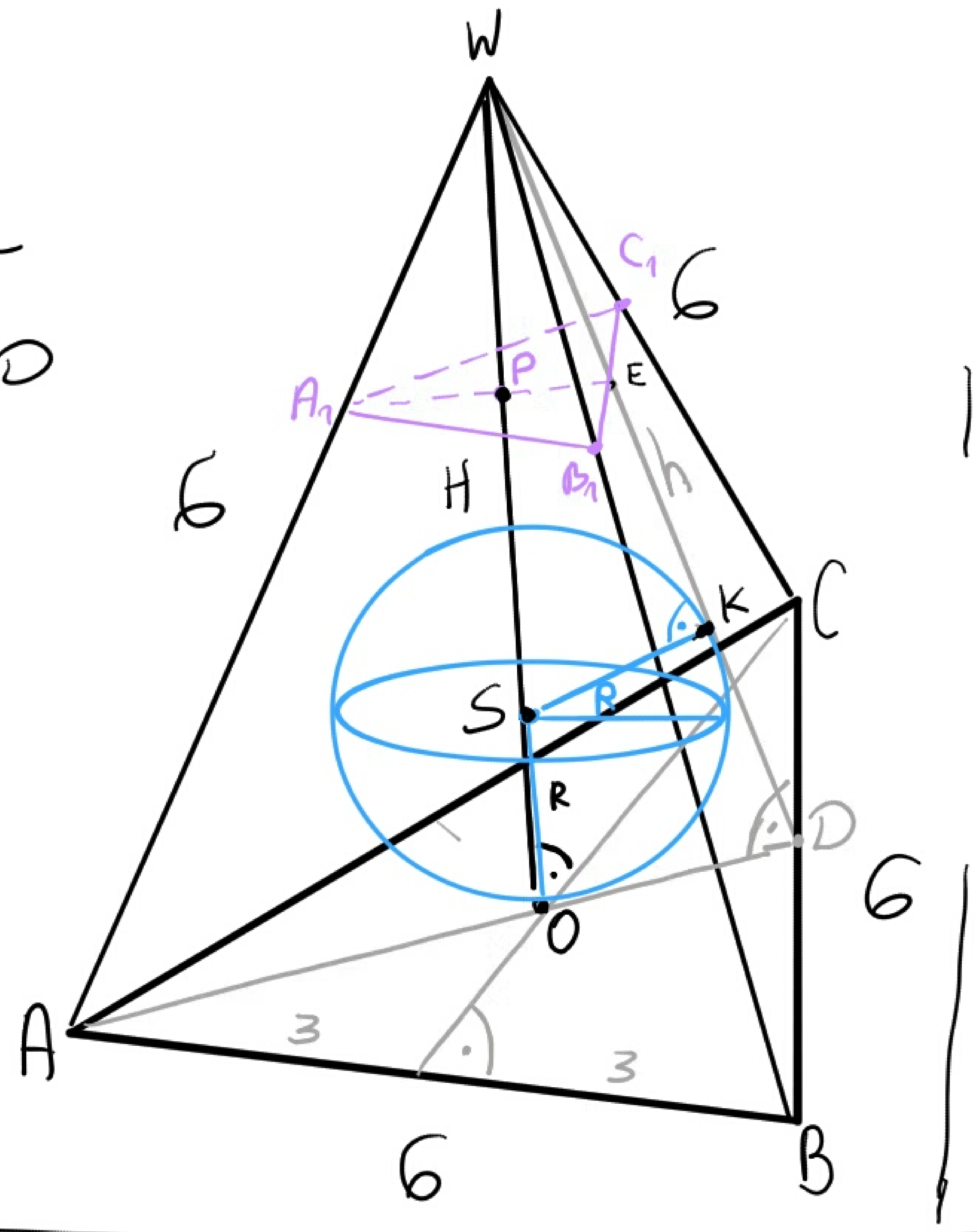
$|A_1B_1| = |B_1C_1| = |A_1C_1| = a_1 > 0$

$\overline{A_1C_1} \parallel \overline{AC}$

$\overline{A_1B_1} \parallel \overline{AB}$ ....

$V = V_{ABCW}$

$V_1 = V_{A_1B_1C_1W} = \frac{8}{27} V$



Sz:

$|SP| = ?$

① $|OA| = \frac{a\sqrt{3}}{3} = 2\sqrt{3} \land |OD| = \frac{a\sqrt{3}}{6} = \sqrt{3}$

więc z ΔAOW: $|OA|^2 + H^2 = a^2 \Rightarrow \underline{H} = \sqrt{a^2 - \frac{1}{3}a^2} = \frac{a\sqrt{6}}{3} = \underline{2\sqrt{6}}$

analogicznie $\Rightarrow \underbrace{H_1 = \frac{a_1\sqrt{6}}{3}}$

② $V_1 = \frac{8}{27} V$

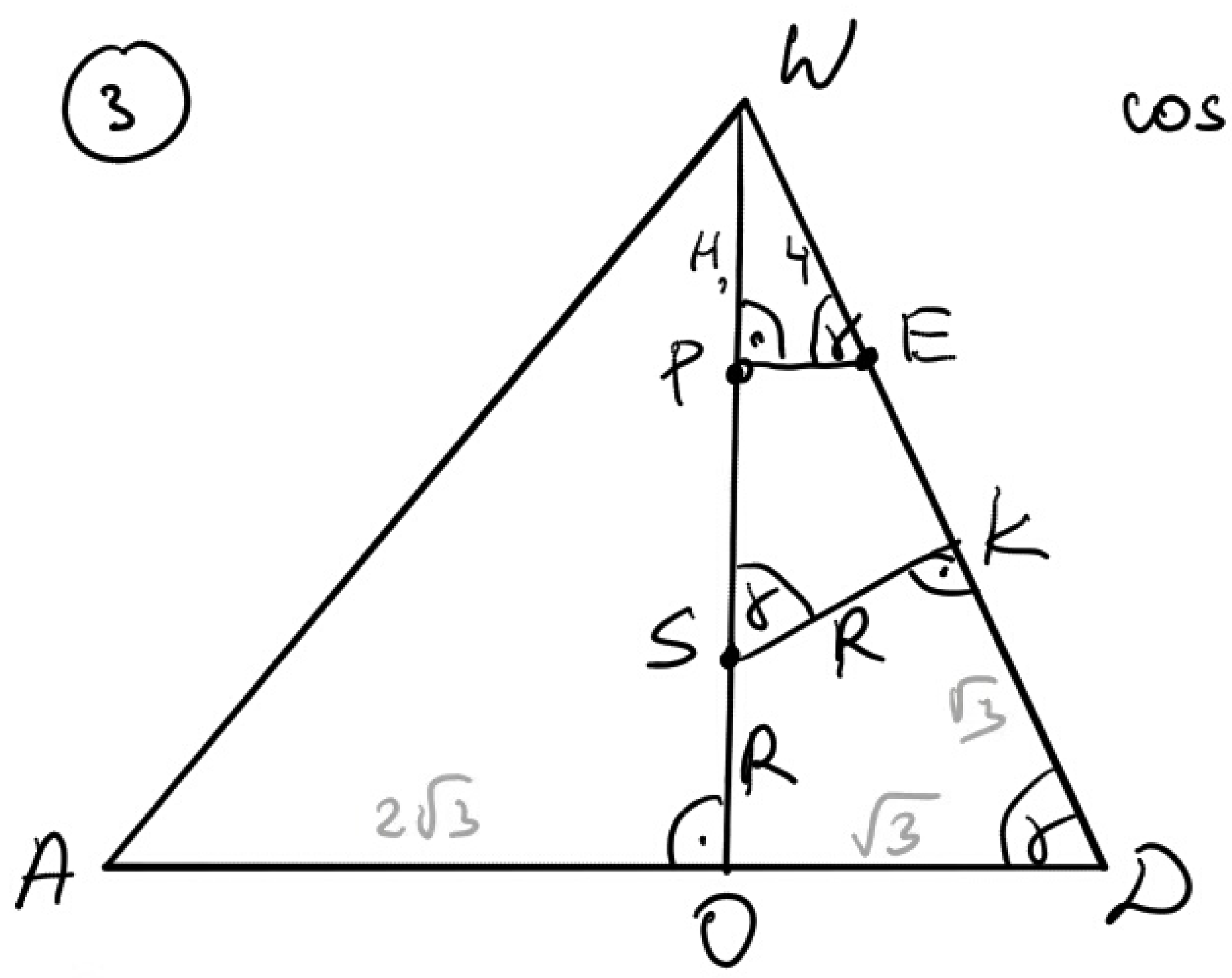
$\frac{1}{3} \cdot \frac{a_1^2\sqrt{3}}{4} \cdot H_1 = \frac{8}{27} \cdot \frac{1}{3} \cdot \frac{6^2\sqrt{3}}{4} \cdot H \quad \Big/ \cdot \frac{4 \cdot 3}{\sqrt{3}}$

$a_1^2 H_1 = \frac{8}{\cancel{27}_{3}} \cdot \cancel{36}^{4} \cdot H \quad \land \quad H = 2\sqrt{6} \quad \land \quad H_1 = \frac{a_1\sqrt{6}}{3}$

$\frac{a_1^3\sqrt{6}}{3} = \frac{8 \cdot 4}{3} \cdot 2\sqrt{6} \quad \Big/ \cdot \frac{3}{\sqrt{6}}$

$a_1^3 = 64 = 4^3 \Rightarrow \underbrace{a_1 = 4} \Rightarrow \underbrace{H_1 = \frac{4}{3}\sqrt{6}}$

③



$\cos\gamma = \frac{|OD|}{|DW|} = \frac{|SK|}{|SW|} \land |OD| = |DK| = \frac{a\sqrt{3}}{6} = \sqrt{3}$

$|DW| = h = \frac{a\sqrt{3}}{2} = 3\sqrt{3}$

$\frac{\sqrt{3}}{\quad} = \frac{R}{H-R}$

$\frac{\cancel{\sqrt{3}}}{3\cancel{\sqrt{3}}} = \frac{R}{2\sqrt{6} - R}$

$3R = 2\sqrt{6} - R$

$4R = 2\sqrt{6} \Rightarrow \underbrace{R = \frac{\sqrt{6}}{2}}$

④ $|SP| = H - R - H_1 = 2\sqrt{6} - \frac{1}{2}\sqrt{6} - \frac{4}{3}\sqrt{6} =$

$= \frac{12}{6}\sqrt{6} - \frac{3}{6}\sqrt{6} - \frac{8}{6}\sqrt{6} = \frac{\sqrt{6}}{6}$

Zad. 10. <4 pkt>

$\cos 2x + 3\cos x = -2 \quad$ dla $\quad x \in \langle 0; 2\pi \rangle$

$2\cos^2 x - 1 + 3\cos x + 2 = 0$

$2\cos^2 x + 3\cos x + 1 = 0 \quad / : 2 \quad$ (I metoda)

$\cos^2 x + \frac{3}{2}\cos x + \frac{1}{2} = 0$

$\left(\cos x + \frac{3}{4}\right)^2 - \frac{9}{16} + \frac{8}{16} = 0$

$\left(\cos x + \frac{3}{4}\right)^2 = \frac{1}{16} \quad / \sqrt{\ }$

$\left|\cos x + \frac{3}{4}\right| = \frac{1}{4}$

$\cos x = -\frac{3}{4} \mp \frac{1}{4} = \begin{cases} -1 \\ -\frac{1}{2} \end{cases} \quad$ dla $\quad k \in C$

$\cos x = -1 \quad \vee \quad \cos x = -\frac{1}{2}$

$x = \pi + 2k\pi \quad \vee \quad x \overset{II}{=} \frac{2\pi}{3} + 2k\pi \quad \vee \quad x \overset{III}{=} \frac{4\pi}{3} + 2k\pi$

dla $x \in \langle 0; 2\pi \rangle$ mamy: $\quad x = \left\{ \frac{2\pi}{3}; \pi; \frac{4\pi}{3} \right\}.$

$2\cos^2 x + 3\cos x + 1 = 0 \quad$ (II metoda – z Δ)

z: $\quad t = \cos x \in \langle -1; 1 \rangle$

$2t^2 + 3t + 1 = 0$

$\Delta_t = 9 - 4 \cdot 2 \cdot 1 = 1 \quad ; \sqrt{\Delta_t} = 1$

$t_1 = \frac{-3-1}{4} = -1 \quad \vee \quad t_2 = \frac{-3+1}{4} = -\frac{1}{2}$

$\cos x = -1 \quad \vee \quad \cos x = -\frac{1}{2}$

$x = \pi + 2k\pi \quad \vee \quad x \overset{II}{=} \frac{2\pi}{3} + 2k\pi \quad \vee \quad x \overset{III}{=} \frac{4\pi}{3} + 2k\pi$

dla $x \in \langle 0; 2\pi \rangle$ mamy: $\quad x = \left\{ \frac{2\pi}{3}; \pi; \frac{4\pi}{3} \right\}.$

Zad. 11 <4pkt>

$P(A) = ?$

$Z = \{1;2;3;4;5;6;7;8\}$

$n = 8$

$k = 3$

ki, P

$\Rightarrow \overline{\overline{\Omega}} = n^k$

A – wylosowano 3 liczby, których iloczyn jest podzielny 4

OZNACZENIA:

$A' = \Omega - A$

NP → l. nieparzyste → $\{1;3;5;7\}$

P → l. parzyste → $\{2;4;6;8\}$

$4|$ → l. podzielne na 4 → $\{4;8\}$

$4\nmid P$ → l. parzyste, niepodzielne na 4 → $\{2;6\}$

① $\overline{\overline{\Omega}} = 8^3 = 512$

(I metoda – z A')

② $$\overline{\overline{A'}} = \underset{\substack{NP\\1\\3\\5\\7}}{4}\;\underset{\substack{NP\\1\\3\\5\\7}}{4}\;\underset{\substack{NP\\1\\3\\5\\7}}{4} + \underset{\substack{NP\\1\\3\\5\\7}}{4}\;\underset{\substack{NP\\1\\3\\5\\7}}{4}\;\underset{\substack{4\nmid P\\2\\6}}{2} \times 3 = 64 + 32 \cdot 3 = 64 + 96 = \underline{\underline{160}}$$

③ $$P(A') = \frac{\overline{\overline{A'}}}{\overline{\overline{\Omega}}} = \frac{160}{512} = \frac{5}{16}$$

④ $$P(A) = 1 - P(A') = \frac{16}{16} - \frac{5}{16} = \frac{11}{16}$$

(II metoda)

② $$\overline{\overline{A}} = \underset{4|}{2}\;\underset{4|}{2}\;\underset{4|}{2} + \underset{4|}{2}\;\underset{4|}{2}\;\underset{\substack{NP \vee 4\nmid P\\1,2,3,5,6,7}}{6} \times 3 + \underset{4|}{2}\;\underset{NP\vee 4\nmid P}{6}\;\underset{NP\vee 4\nmid P}{6} \times 3 +$$

$$+ \underset{P}{2}\;\underset{P}{2}\;\underset{P}{2} + \underset{P}{2}\;\underset{P}{2}\;\underset{NP}{4} \times 3 =$$

$$= 8 + 24 \times 3 + 72 \times 3 + 8 + 16 \times 3 = \underline{\underline{352}}$$

③ $$P(A) = \frac{\overline{\overline{A}}}{\overline{\overline{\Omega}}} = \frac{352}{512} = \frac{11}{16}$$

Zad. 12 (5 pkt)

$m = ?$

$4x^2 - 6mx + (2m+3)(m-3) = 0$

$$\begin{cases} ① & \Delta x > 0 \\ ② & (4x_1 - 4x_2 - 1)(4x_1 - 4x_2 + 1) < 0 \\ ③ & x_1 < x_2 \quad \Rightarrow \quad x_1 - x_2 < 0 \end{cases}$$

① $36m^2 - 4 \cdot 4(2m^2 - 3m - 9) > 0 \quad /:4$

$9m^2 - 8m^2 + 12m - 36 > 0$

$m^2 + 12m + 36 > 0$

$(m+6)^2 > 0 \quad \Rightarrow \quad z_1: \underline{m \in \mathbb{R} \setminus \{-6\}}$

② $[4(x_1 - x_2) - 1] \cdot [4(x_1 - x_2) + 1] < 0$

$16(x_1 - x_2)^2 - 1 < 0$

$16(x_1 - x_2)^2 < 1 \quad /:16$ $\quad \boxed{\text{I metoda:}}$

$x_1^2 + x_2^2 - 2x_1x_2 + 2x_1x_2 - 2x_1x_2 < \frac{1}{16}$

$(x_1 + x_2)^2 - 4x_1x_2 < \frac{1}{16}$

$\left(\frac{\cancel{6}^{3}m}{\cancel{4}_{2}}\right)^2 - \cancel{4} \cdot \frac{2m^2 - 3m - 9}{\cancel{4}} < \frac{1}{16}$

$\frac{9}{4}m^2 - 2m^2 + 3m + 9 < \frac{1}{16} \quad /\cdot 4$

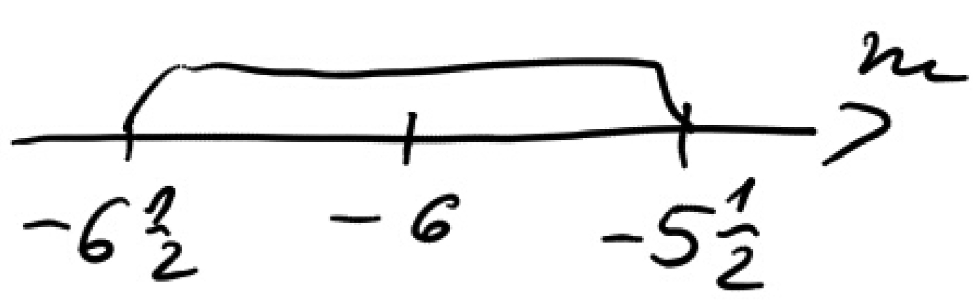
$9m^2 - 8m^2 + 12m + 36 < \frac{1}{4}$

$m^2 + 12m + 35\frac{1}{2} < 0$

$(m+6)^2 - 36 + 35\frac{3}{4} < 0$

$(m+6)^2 < \frac{1}{4} \quad /\sqrt{}$

$|m+6| < \frac{1}{2}$



$z_2: \underline{m \in \left(-6\frac{1}{2}; -5\frac{1}{2}\right)}$

Odp: $z_1 \cap z_2: \; m \in \left(-6\frac{1}{2}; -5\frac{1}{2}\right) \setminus \{-6\}$

Zad. 12 (5pkt)

$m = ?$

$$4x^2 - 6mx + (2m+3)(m-3) = 0$$

$$\begin{cases} ① \ \Delta_x > 0 \\ ② \ (4x_1 - 4x_2 - 1)(4x_1 - 4x_2 + 1) < 0 \\ ③ \ x_1 < x_2 \implies x_1 - x_2 < 0 \end{cases}$$

① $36m^2 - 4\cdot 4(2m^2 - 3m - 9) > 0 \quad /:4$

$9m^2 - 8m^2 + 12m - 36 > 0$

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$(m+6)^2 > 0 \implies Z_1: \ \underline{m \in \mathbb{R} \setminus \{-6\}}$

② $[4(x_1 - x_2) - 1]\cdot[4(x_1 - x_2) + 1] < 0$

$16(x_1 - x_2)^2 - 1 < 0$

$16(x_1 - x_2)^2 < 1 \quad /:16$

$(x_1 - x_2)^2 < \frac{1}{16} \quad /\sqrt{\ }$

$|x_1 - x_2| < \frac{1}{4} \quad \wedge \quad ③ \ x_1 - x_2 < 0$, więc:

$-(x_1 - x_2) < \frac{1}{4} \quad /\cdot(-1)$

$x_1 - x_2 > -\frac{1}{4}$

II metoda :

oraz

$$x_1 - x_2 = \frac{-b - \sqrt{\Delta}}{2a} - \frac{-b + \sqrt{\Delta}}{2a} = \frac{-b - \sqrt{\Delta} + b - \sqrt{\Delta}}{2a} = \frac{-2\sqrt{\Delta}}{2a} = \frac{-(b^2 - 4ac)}{a} = \frac{4ac - b^2}{a}$$

$$\boxed{x_1 - x_2 = 4c - \frac{b^2}{a}}$$

④

$4\cdot(2m+3)(m-3) - \frac{\cancel{36}^{\,9}m^2}{\cancel{4}_{\,1}} > -\frac{1}{4}$

$4(2m^2 - 3m - 9) - 9m^2 + \frac{1}{4} > 0$

$8m^2 - 12m - 36 - 9m^2 + \frac{1}{4} > 0$

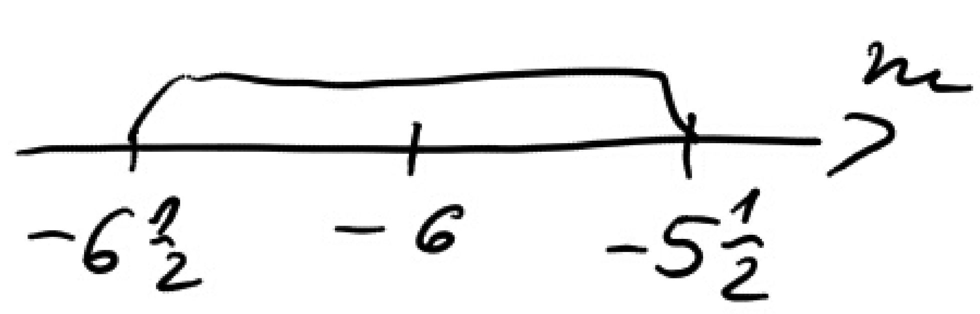
$-m^2 - 12m - 35\frac{3}{4} > 0 \quad /(-1)$

$m^2 + 12m + 35\frac{3}{4} < 0$

$(m+6)^2 - 36 + 35\frac{3}{4} < 0$

$(m+6)^2 < \frac{1}{4} \quad /\sqrt{\ }$

$|m+6| < \frac{1}{2}$



$Z_2: \ \underline{m \in \left(-6\frac{1}{2}; -5\frac{1}{2}\right)}$

Odp: $Z_1 \cap Z_2: \ m \in \left(-6\frac{1}{2}; -5\frac{1}{2}\right) \setminus \{-6\}$

Zad. 13 (5pkt)

$O_1: (x-a)^2+(y-b)^2=r^2$?

I metoda:

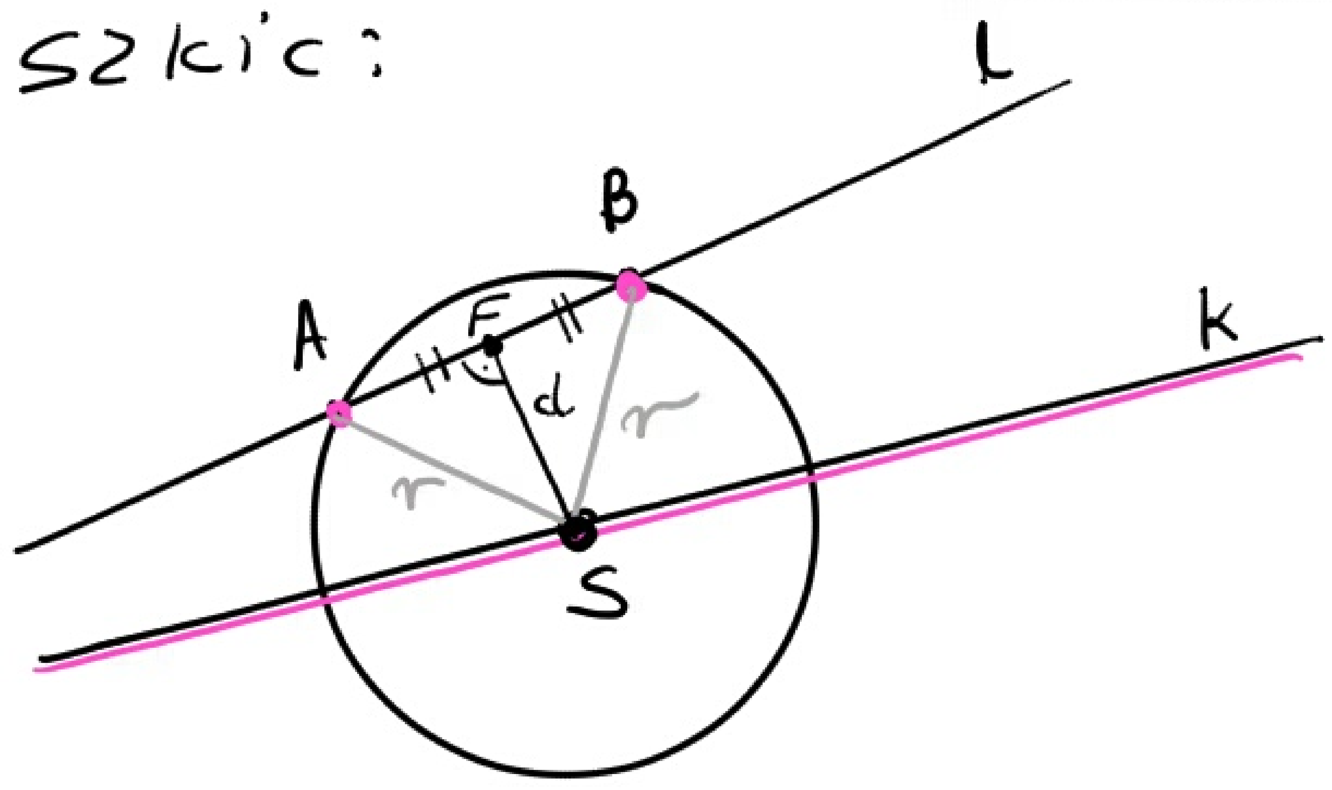
$A=(-5;3) \in O_1$

$B=(0;6) \in O_1$

$S=(a,b) \in k$

$k: x-3y+1=0$

szkic:



① $S \in k \wedge k: x=3y-1$

$S=(3y_s-1; y_s) = (\underbrace{3b-1}_{a}; b)$

② $\overrightarrow{SB} = [3b-1; b-6]$

$r^2 = |SB|^2 = (3b-1)^2+(b-6)^2 =$

$= 9b^2-6b+1+b^2-12b+36 = \underline{10b^2-18b+37}$

③ $O_1: (x-a)^2+(y-b)^2=r^2$

$(x-3b+1)^2+(y-b)^2 = 10b^2-18b+37$

④ $A \in O_1$: $(-5-3b+1)^2+(3-b)^2 = 10b^2-18b+37$

$(3b+4)^2+(3-b)^2 = 10b^2-18b+37$

$\cancel{9b^2}+24b+16+9-6b+\cancel{b^2} = \cancel{10b^2}-18b+27$

$36b = 12 \quad /:36$

$b=\frac{1}{3} \Rightarrow \underline{S=(0;\frac{1}{3})}$

$r^2 = 10\cdot\frac{1}{9} - \cancel{18}^{6}\cdot\frac{1}{\cancel{3}}+37 = 1\frac{1}{9}+31 = 32\frac{1}{9} = \frac{289}{9} = \left(\frac{17}{3}\right)^2$

⑤ $O_1: x^2+\left(y-\frac{1}{3}\right)^2 = \frac{289}{9}$

Zad. 13 (5pkt)

$O_1: (x-a)^2+(y-b)^2=r^2$?

II metoda:

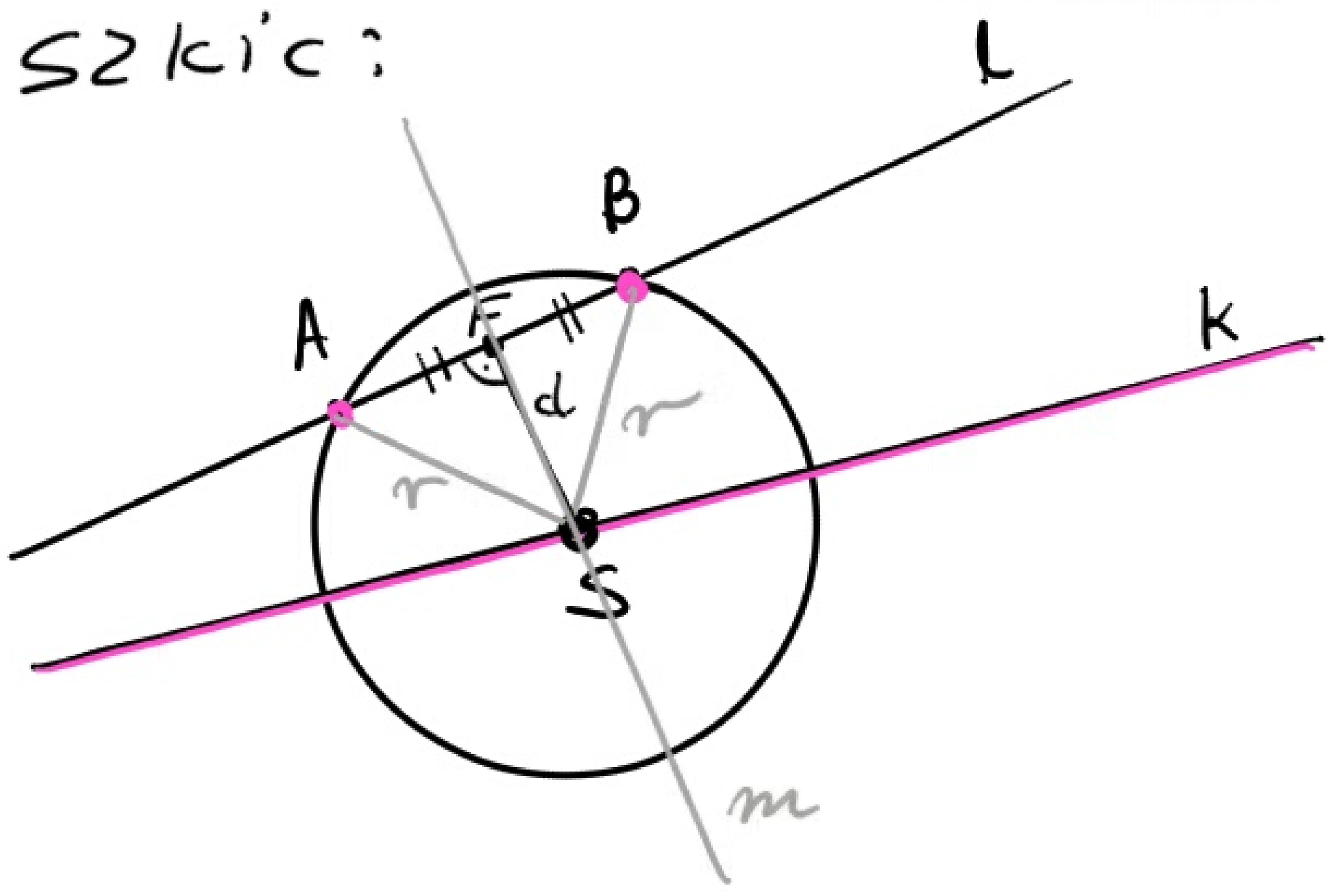
$A=(-5;3)\in O_1$

$B=(0;6)\in O_1$

$S=(a,b)\in k$

$k: x-3y+1=0$

szkic:



① $F=S_{AB}=\left(-\frac{5}{2};\frac{9}{2}\right)$

② $l=l_{AB}$:

$\begin{cases} -5a+b=3 \\ 0\cdot a+b=6 \end{cases} \Rightarrow \begin{cases} -5a=-3 \quad /:(-5) \\ b=6 \end{cases}$

$l: y=\frac{3}{5}x+6$

③

$m=l_{FS}\perp l \Rightarrow m: y=-\frac{5}{3}x+n$

$F\in m \Rightarrow$

$\frac{9}{2}=-\frac{5}{3}\cdot\left(-\frac{5}{2}\right)+n$

$\frac{27}{6}=\frac{25}{6}+n$

$n=\frac{1}{3} \Rightarrow m: y=-\frac{5}{3}x+\frac{1}{3}$

④ $S\in(k\cap m)$:

$\begin{cases} x-3y+1=0 \\ y=-\frac{5}{3}x+\frac{1}{3} \end{cases} \Rightarrow x-3\left(-\frac{5}{3}x+\frac{1}{3}\right)+1=0$

$x+5x-1+1=0$

$6x=0 \quad /:6$

$\begin{cases} x=0 \\ y=\frac{1}{3} \end{cases} \Rightarrow S=\left(0;\frac{1}{3}\right)$

⑤ $\overrightarrow{SB}=\left[0;6-\frac{1}{3}\right]$

$r^2=|SB|^2=0^2+\left(5\frac{2}{3}\right)^2=\left(\frac{17}{3}\right)^2=\frac{289}{9}$

Odp: $O_1: x^2+\left(y-\frac{1}{3}\right)^2=\frac{289}{9}$

Zad. 14 <6 pkt>

$(a, b, c) \Rightarrow$ c.a

$a+b+c = 27$

$(a-2; b; 2c+1) \Rightarrow$ c.g

$a, b, c = ?$

$$\begin{array}{l} \boxed{1} \\ \boxed{2} \\ \boxed{3} \end{array} \begin{cases} c-b = b-a \\ a+b+c = 27 \\ b^2 = (a-2)(2c+1) \end{cases} \qquad \begin{cases} \underbrace{c = 2b-a} \\ \cancel{a}+b+2b-\cancel{a} = 27 \\ b^2 = (a-2)\cdot[2(2b-a)+1] \end{cases}$$

$$\begin{array}{l} \boxed{2} \\ \boxed{3} \end{array} \begin{cases} 3b = 27 \quad /:3 \quad \Rightarrow \quad \underbrace{b = 9} \\ 9^2 = (a-2)(4\cdot 9 - 2a + 1) \end{cases}$$

$\boxed{3}$ $81 = (a-2)(37-2a)$

$81 = \underline{37a} - 2a^2 - 74 + \underline{4a}$

$2a^2 - 41a + 155 = 0 \quad /:2$

$a^2 - \frac{41}{2}a + \frac{155}{2} = 0$

$\left(a - \frac{41}{4}\right)^2 - \frac{1681}{16} + \frac{8\cdot 155}{16} = 0$

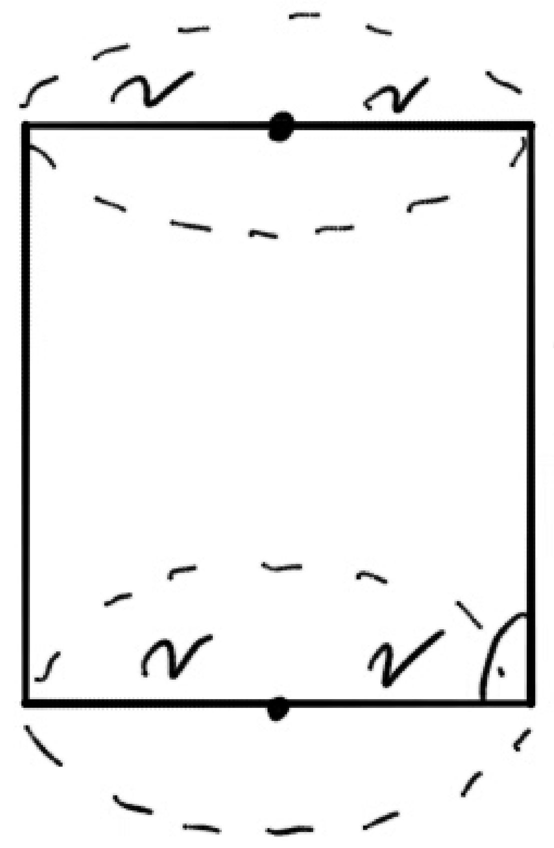
$\left(a - \frac{41}{4}\right)^2 = \frac{441}{16} \quad /\sqrt{\ }$

$\left|a - \frac{41}{4}\right| = \frac{21}{4} \Rightarrow a = \frac{41}{4} \mp \frac{21}{4} = \begin{cases} \frac{20}{4} = 5 \\ \frac{62}{4} = \frac{31}{2} = 15\frac{1}{2} \end{cases}$

Odp:

$$\begin{cases} a = 5 \\ b = 9 \\ c = 13 \end{cases} \quad \vee \quad \begin{cases} a = 15\frac{1}{2} \\ b = 9 \\ c = 2\frac{1}{2} \end{cases}$$

Zad. 15 ⟨7 pkt.⟩



D:

$P_c = P$

$V = max$

Sz:

$H = ?$

$r = ?$

$V = ?$

z: $r, V, H, P > 0$

(1) $2\pi r \cdot H + 2\pi r^2 = P$

$2\pi r (H + r) = P \quad / : (2\pi r)$

$H + r = \frac{P}{2\pi r} \implies \underbrace{H = \frac{P}{2\pi r} - r}$

(2) $H > 0 \implies \frac{P}{2\pi r} - r > 0 \quad / \cdot 2\pi r$

$P - 2\pi r^2 > 0$

$2\pi r^2 < P \quad / : (2\pi)$

$r^2 < \frac{P}{2\pi} \quad / \sqrt{\ } \quad \wedge \quad r > 0$

$r < \sqrt{\frac{P}{2\pi}} = \frac{\sqrt{P}}{\sqrt{2\pi}} = \frac{\sqrt{2P\pi}}{2\pi}$

$\underbrace{D: \ r \in \left(0; \frac{\sqrt{2P\pi}}{2\pi}\right)}$

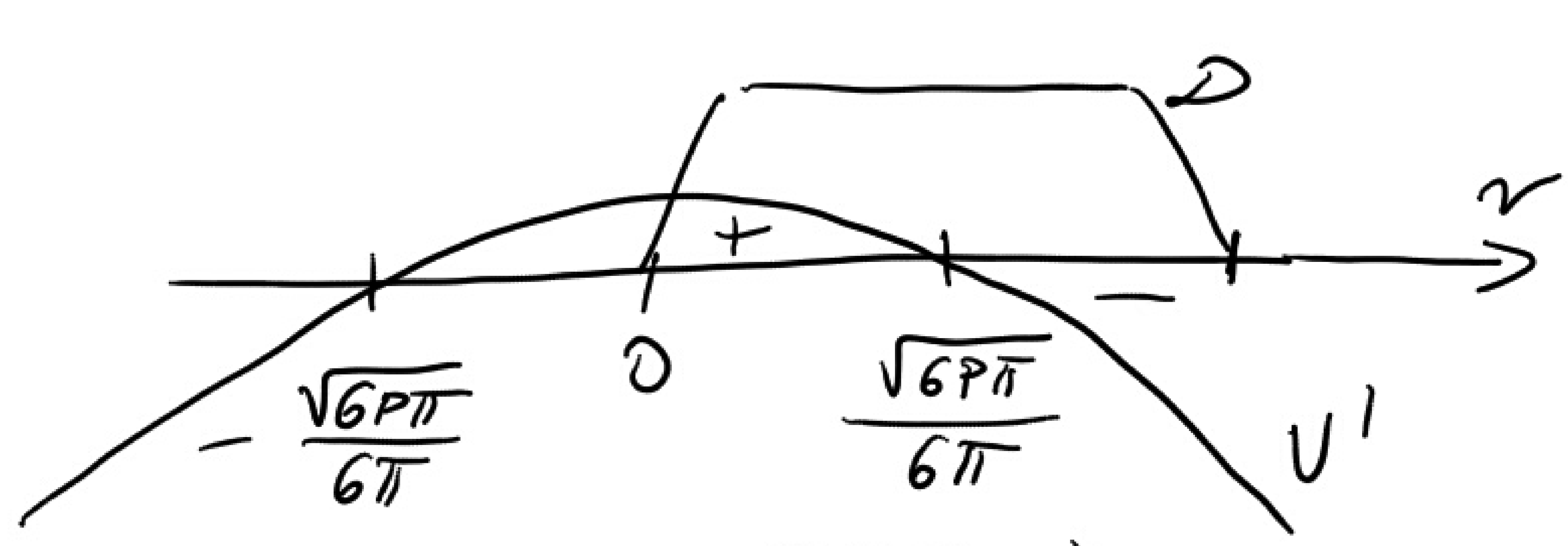
(3) $V = \pi r^2 \cdot H = \pi r^2 \cdot \left(\frac{P}{2\pi r} - r\right) = \frac{1}{2} \cdot P r - \pi r^3$

$\underbrace{V(r) = -\pi r^3 + \frac{1}{2} \cdot P \cdot r} \quad \wedge \quad D = \left(0; \frac{\sqrt{2\pi P}}{2\pi}\right)$

(4) $V'(r) = -3\pi r^2 + \frac{1}{2}P = -3\pi \cdot \left(r^2 - \frac{P}{6\pi}\right) =$

$= -3\pi \left(r - \sqrt{\frac{P}{6\pi}}\right)\left(r + \sqrt{\frac{P}{6\pi}}\right) = -3\pi \left(r - \frac{\sqrt{6P\pi}}{6\pi}\right)\left(r + \frac{\sqrt{6P\pi}}{6\pi}\right)$

$r = \frac{\sqrt{6P\pi}}{6\pi} = \frac{\sqrt{P}}{\sqrt{6\pi}} \quad \Longleftarrow$



V: ↗ max lok.=glob. ↘

$H = \frac{P}{2\pi r} - r$

$\underbrace{H} = \frac{\cancel{P}^{\sqrt{P}}}{2\pi} \cdot \frac{\sqrt{6\pi}}{\cancel{\sqrt{P}}} - \frac{\sqrt{6\pi} \cdot \sqrt{P}}{6\pi} = \frac{3 \cdot \sqrt{6P\pi} - \sqrt{6P\pi}}{6\pi} = \frac{2\sqrt{6P\pi}}{6\pi} = \underbrace{\frac{\sqrt{6P\pi}}{3\pi}}$

$\underbrace{V} = \pi r^2 H = \cancel{\pi} \cdot \frac{P}{6\pi} \cdot \frac{\sqrt{6P\pi}}{3\cancel{\pi}} = \underbrace{\frac{P\sqrt{6P\pi}}{18\pi}}$