Matura podstawa 2020

Zad. 1

$$x^2 - 6x + 9 = (x-3)^2 = \left\{ \begin{array}{l} \text{albo} \\ x = \sqrt{3} + 3 \end{array} \right\} =$$

$$= (\sqrt{3} + 3 - 3)^2 = \sqrt{3}^2 = 3$$

B

Zad. 2

$$\frac{2^{50} \cdot 3^{40}}{36^{10}} = \frac{2^{50} \cdot 3^{40}}{(2^2 \cdot 3^2)^{10}} = \frac{2^{50} \cdot 3^{40}}{2^{20} \cdot 3^{20}} =$$

$$= 2^{30} \cdot 3^{20}$$

C

Zad. 3

$$\log_5 \sqrt{125} = \log_5 \sqrt{5^3} = \log_5 5^{\frac{3}{2}} = \frac{3}{2}$$

D

Zad. 4

$$y = 0{,}8x - 100\%.$$

$$x - y = 0{,}2x \quad - \quad a = ?$$

$$a = \frac{0{,}2 \cdot 100\%.}{0{,}8x} = 25\%$$

A

Zad. 5

$$3(1-x) > 2(3x-1) - 12x$$
$$3 - 3x > 6x - 2 - 12x$$
$$3x > -5 \quad /:3$$
$$x > -\frac{5}{3}$$
$$x \in \left(-\frac{5}{3}; \infty\right)$$

A

Zad. 6

$$x(x-3)(x+2) = 0$$
$$x_1 = 0, \; x_2 = 3, \; x_3 = -2$$
$$x_1 + x_2 + x_3 = 1$$

B

Zad. 7-9: $f(x) = a(x-1)(x-3)$

$W = (2; 1) \Rightarrow$ wierzchołek $f(x)$

$\Downarrow$

$f(2) = 1$

$a(2-1)(2-3) = 1$

$-a = 1 \quad /(-1)$

$a = -1$

Zad. 7

D

Zad. 8

$f_{max} = 1$ dla $x \in \langle 1; 4 \rangle$

C

Zad. 9

os. symetrii: $x = 2$

B

Zad. 10

$$x(x-2) = (x-2)^2$$
$$x(x-2) - (x-2)^2 = 0$$
$$(x-2)[x-(x-2)] = 0$$
$$(x-2)\cdot(-2) = 0$$
$$-2(x-2) = 0 \quad /:(-2)$$
$$x-2 = 0$$
$$x = 2$$

1 rozwiązanie

(B)

Zad. 11

$f(x) = ax + b$

z wykresu: $f \searrow \Rightarrow a < 0$
$b = 1 \Rightarrow b > 0$
$\Rightarrow ab < 0$

(D)

Zad. 12

$f(x) = 4^{-x} + 1$

$f(\frac{1}{2}) = 4^{-\frac{1}{2}} + 1 = (\frac{1}{4})^{\frac{1}{2}} + 1 = \sqrt{\frac{1}{4}} + 1$

$= \frac{1}{2} + 1 = \frac{3}{2}$

(B)

Zad. 13

$l_1: y = (m-2)x$
$l_2: y = \frac{3}{4}x + 7$ | $m = ?$

(1) $l_1 \parallel l_2$

(1) $m-2 = \frac{3}{4} \Rightarrow m = \frac{3}{4} + 2 = \frac{11}{4}$

(C)

Zad. 14 $a_n = 2n^2$ | $a_5 - a_4 = ?$

$a_5 - a_4 = 2 \cdot 5^2 - 2 \cdot 4^2 = 50 - 32 = \underline{18}$ (D)

Zad. 15 $a_n = a_1 + (n-1) \cdot r$ | $S_4 = a_1 + a_2 + a_3 + a_4 = ?$

(1) $a_4 = 3$

$r = 5$

(1) $a_1 + 3r = 3$

$a_1 = 3 - 3 \cdot 5 = -12$

(2) $S_4 = \frac{2 \cdot a_1 + 3r}{2} \cdot 4^2 = [2 \cdot (-12) + 3 \cdot 5] \cdot 2$

$= 2(-24 + 15) = 2 \cdot (-9) = \underline{-18}$

(C)

Zad. 16 $f(x) = 3x + b$ | $b = ?$

(1) $A = (\frac{1}{3}; -1) \in f(x)$

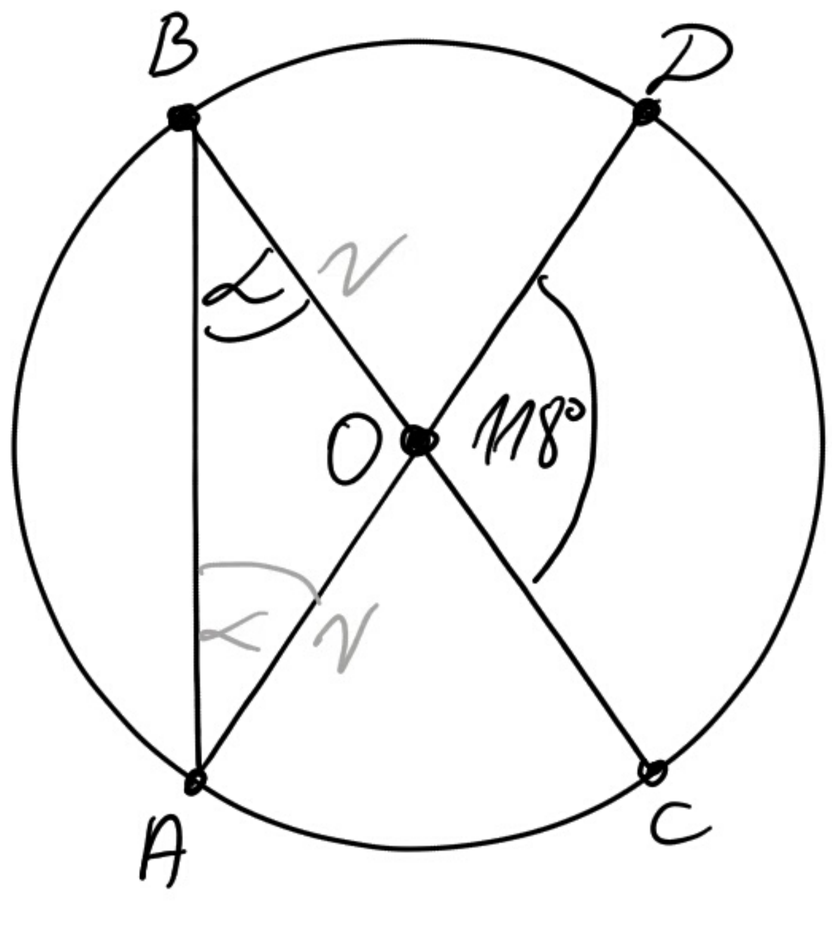
(1) $f(\frac{1}{3}) = -1$

$3 \cdot \frac{1}{3} + b = -1$

$b = -2$

(D)

Zad. 17



$\alpha = ?$

(1) $|\sphericalangle AOB| = 118^\circ = |\sphericalangle DOC|$

(2) $\Delta AOB \rightarrow$ równoramienny

$|AO| = |OB| = r$

wtedy

$|\sphericalangle ABO| = |\sphericalangle BAO| = \alpha$

(3) $2\alpha + 118^\circ = 180^\circ$

$2\alpha = 62^\circ \quad |:2$

$\alpha = 31^\circ$

D

Zad. 18

$A = (3; -2) \in f$

$B = (-1; 6) \in f$

$f(x) = ax + b = ?$

(1) $\begin{cases} A: 3a + b = -2 \\ B: -a + b = 6 \end{cases}$ —

$4a = -8 \quad |:4$

$a = -2$

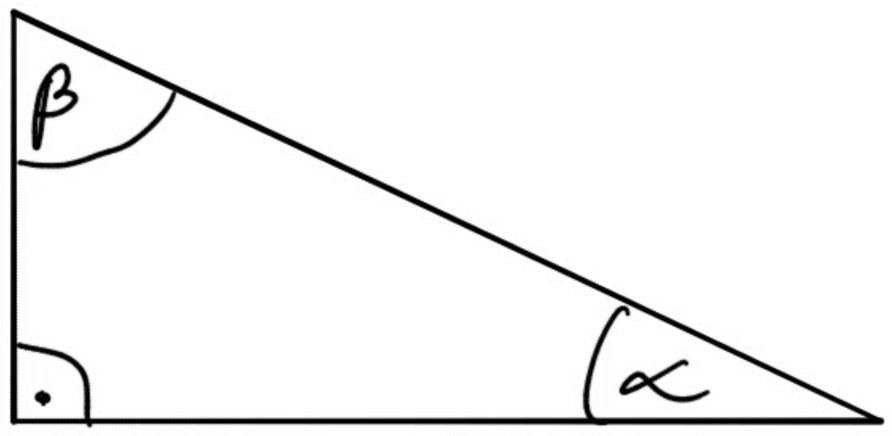
(2) $b = 6 + (-2)$

$b = 4$

(3) $f(x) = -2x + 4$

A

Zad. 19



$2\cos\alpha - \sin\beta = ?$

$$2\cos\alpha - \sin\beta = \begin{cases} \sin\beta = \\ = \cos\alpha \end{cases} \Big\} = 2\cos\alpha - \cos\alpha = \underline{\underline{\cos\alpha}}$$

(B)

Zad. 20

$A = (-3; 5)$

$A \xrightarrow{S(0;0)} B \quad | \quad |AB| = ?$

$B = (3; -5)$

$\overrightarrow{AB} = [6; -10]$

$|AB| = \sqrt{36 + 100} = \sqrt{136} = \underline{\underline{2\sqrt{34}}}$

(A)

Zad. 21

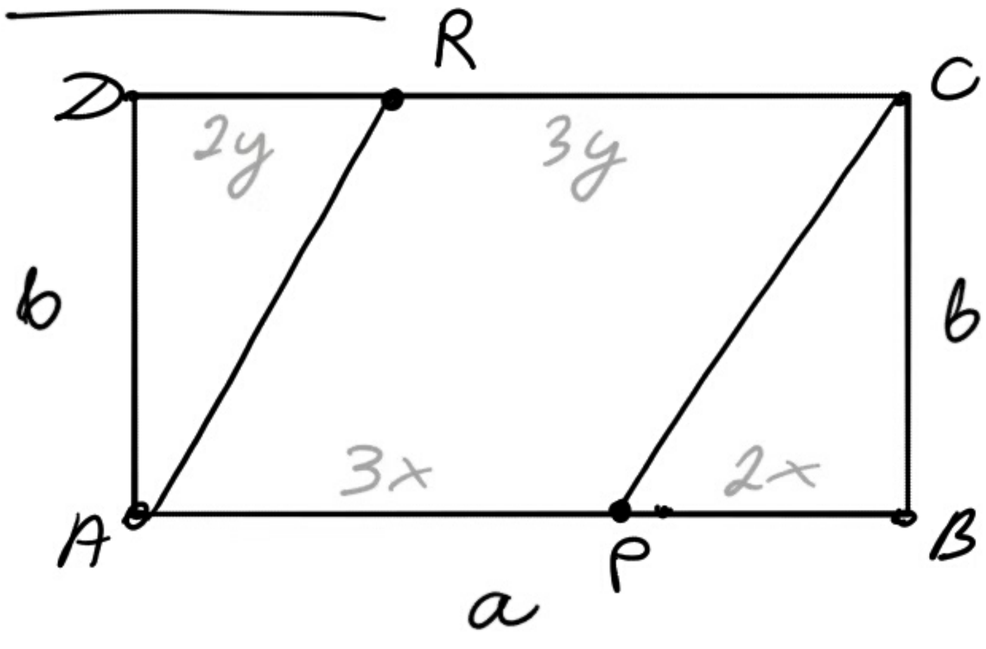
$\Omega = \{1; 3; 5; 7; 9\}$.

$n = 5$ i $k = 2$ i BP $\quad | \quad \overline{\overline{\Omega}} = ?$

$\overline{\overline{\Omega}} = \underline{5} \cdot \underline{4} = \underline{\underline{20}}$

(C)

Zad. 22



(2) $P_{ABCD} = 90$

(1) $\frac{|AP|}{|PB|} = \left|\frac{CR}{RD}\right| = \frac{3}{2}$

$a = |AB|$

$P_{APCR} = P = ?$

(1) $2y + 3y = 3x + 2x \Rightarrow \underline{a = 5x} = 5y$

$5y = 5x$

$\underline{y = x}$

(2) $a \cdot b = 90$

$5x \cdot b = 90 \quad / \cdot \frac{3}{5}$

$3x \cdot b = 90 \cdot \frac{3}{5}$

$P_{APCR} = 3x \cdot b = \frac{270}{5} = \underline{54}$

C

Zad. 23

(2) $2, 3, a, 8$

$2 < 3 < a < 8 \Rightarrow Me = M_1$

(1) $5, 3, 6, 8, 2 \Rightarrow Me = M_2$

$M_1 = M_2$

$a = 2.$

(1) $2, 3, 5, 6, 8 \Rightarrow M_2 = 5$

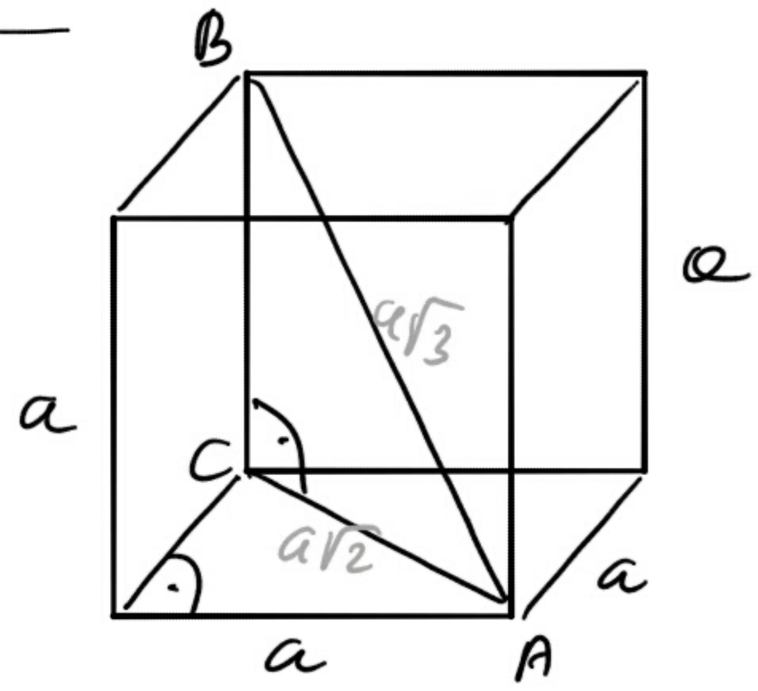
(2) $\frac{3 + a}{2} = 5 \quad / \cdot 2$

$a + 3 = 10$

$\underline{a = 7}$

A

Zad. 24



(1) $|AC| = 4\sqrt{3}$ | $P_c = ?$

(1) $a\sqrt{3} = 4\sqrt{3}$

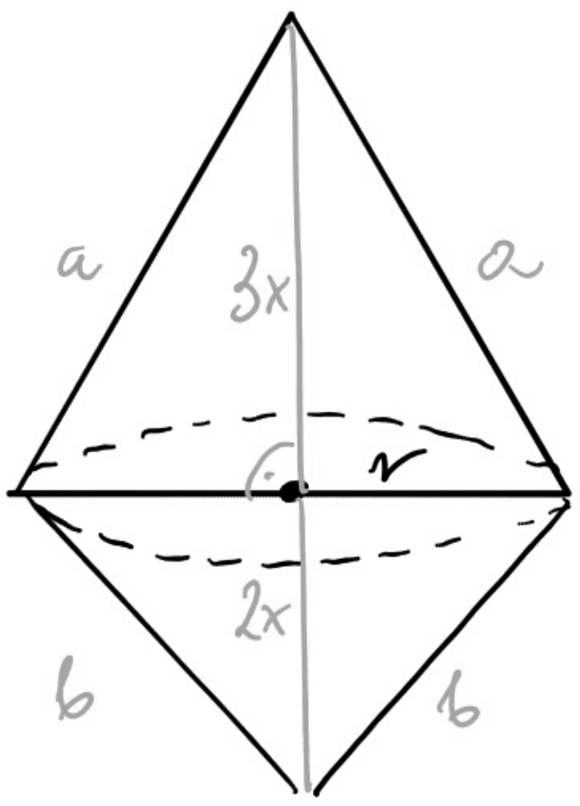
$\underline{a = 4}$

(2) $P_c = 6 \cdot a^2 = 6 \cdot 16$

$\underline{P_c = 96}$

(A)

Zad. 25



$V_1 = \frac{1}{3}\pi r^2 \cdot 3x$

(2) $V_2 = \frac{1}{3}\pi r^2 \cdot 2x = 12\ cm^3$

$V_1 + V_2 = V$

$V = ?$

(1) $V = \frac{1}{3}\pi r^2 (\overbrace{3x + 2x}^{5x})$

$V = \frac{5}{3}\pi r^2 x$

(2) $\frac{2}{3}\pi r^2 x = 12 \quad / \cdot \frac{5}{2}$

$\frac{5}{3}\pi r^2 x = 6 \cdot 5$

$\underline{V = 30\ cm^2}$

(B)

Zad. 26 <2p>

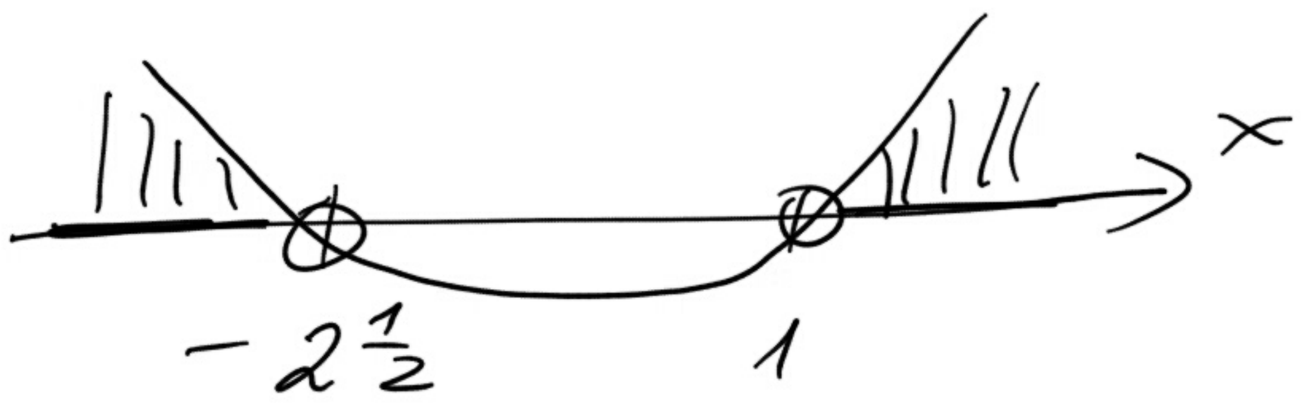
$$2(x-1)(x+3) > x-1$$

$$2(x-1)(x+3) - (x-1) > 0$$

$$(x-1)\left[2(x+3) - 1\right] > 0$$

$$(x-1)(2x+5) > 0$$

$$2(x-1)\left(x+\frac{5}{2}\right) > 0$$



Odp: $x \in (-\infty; -2\frac{1}{2}) \cup (1; \infty)$

Zad. 27 <2 pkt.>

$$(x^2-1)(x^2-2x) = 0$$

$$x \cdot (x-1)(x+1) \cdot (x-2) = 0$$

Odp: $x = \{-1; 0; 1; 2\}$.

Zad. 28 <2 plet.>

Z: $a \neq b$
$a, b \in \mathbb{R}$

T:
$a(a-2b)+2b^2 > 0$

D:
$a(a-2b)+2b^2 > 0$
$\Updownarrow$
$a^2 - 2ab + b^2 + b^2 > 0$
$\Updownarrow$
$(a-b)^2 + b^2 > 0$ $\wedge$ $a \neq b$

$(a-b)^2 > 0$ (bo $a \neq b$), $b^2 \geq 0$

> 0

> 0

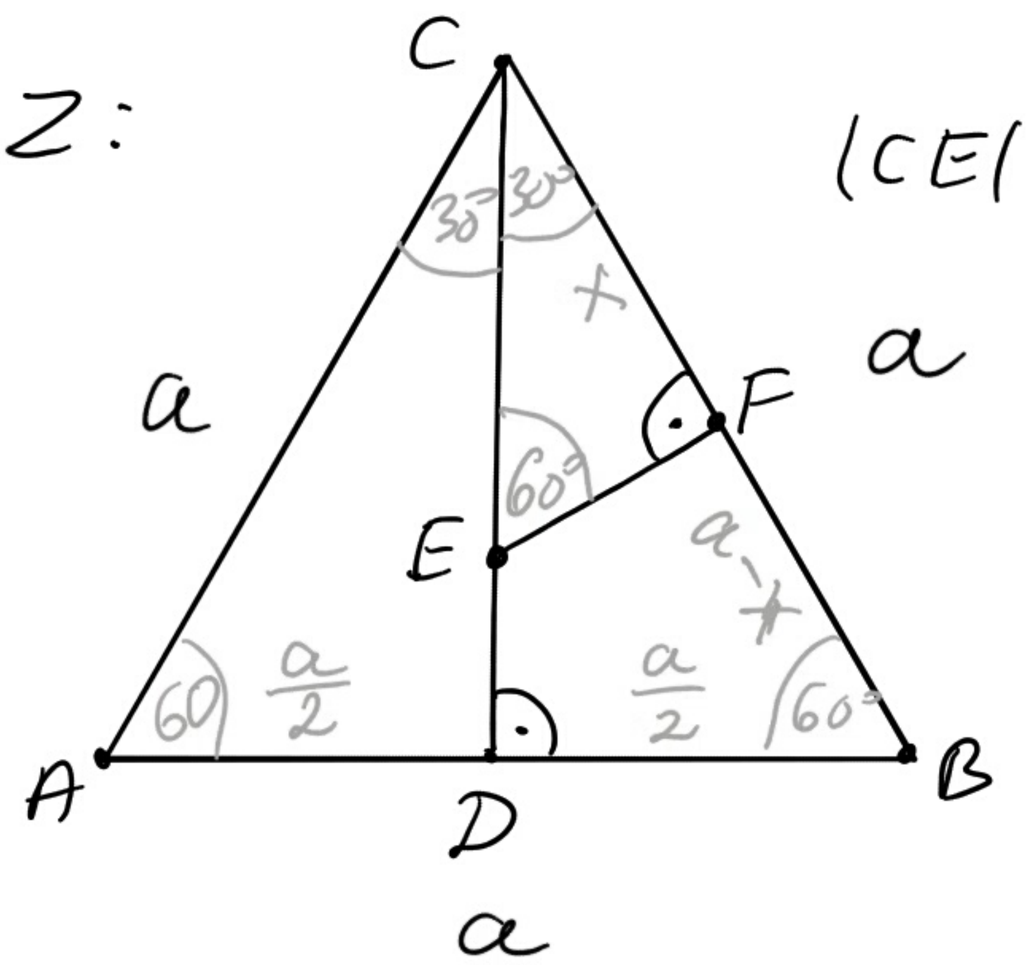
$\Downarrow$
$\begin{cases} \text{dla } a \neq b \wedge a, b \in \mathbb{R} \\ a(a-2b)+2b^2 > 0 \end{cases}$

c.n.d.

Zad. 29. <2 pkt.>

Z: $|CE| = \frac{3}{4}|CD|$

T: $\underbrace{|CF|}_{x} = \frac{9}{16}\underbrace{|CB|}_{a}$



D: (1) $a = |AB| = |AC| = |BC|$

$|CD| = \frac{a\sqrt{3}}{2} \Rightarrow \frac{3}{4}|CD| = \frac{3\sqrt{3}a}{8} = |CE|$

$\frac{1}{4}|CD| = \frac{a\sqrt{3}}{8} = |DE|$

(2) $\triangle EFC \overset{kkk}{\sim} \triangle CDB$ $(30°, 60°, 90°)$

$$\frac{|CE|}{|CB|} = \frac{|CF|}{|CD|} \Rightarrow \frac{\frac{3\sqrt{3}a}{8}}{a} = \frac{x}{\frac{a\sqrt{3}}{2}}$$

$$ax = \frac{3\sqrt{3}a}{8} \cdot \frac{a\sqrt{3}}{2} \quad /: a > 0$$

$$x = \frac{9}{16}a$$

$$|CF| = \frac{9}{16}|CB| \qquad \text{c.n.d.}$$

Zad. 30 <2 pkt.>

$\Omega = \{1; 2; 3; 4; 5; 6\}$

$n = 6$

$k = 2$

ki, p

A - min. raz wypadnie „5"

P(A) = ?

(1) $\overline{\overline{\Omega}} = 6 \cdot 6 = 36$

(2) A' - nie wypadnie „5"

$\overline{\overline{A'}} = 5 \cdot 5 = 25$

$\overline{\overline{A}} = \overline{\overline{\Omega}} - \overline{\overline{A'}} = 36 - 25 = 11$

(3) $P(A) = \frac{\overline{\overline{A}}}{\overline{\overline{\Omega}}} = \frac{11}{36}$

Zad. 31 <2 pkt.>

$\alpha \in (0°; 90°)$

(1) $\frac{2\sin\alpha + 3\cos\alpha}{\cos\alpha} = 4$ | $tg\alpha = ?$

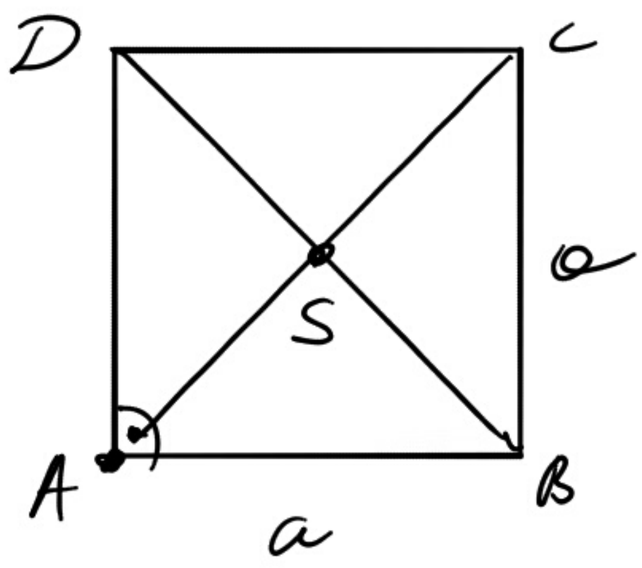
(1) $2 \cdot \frac{\sin\alpha}{\cos\alpha} + 3 \cdot \frac{\cos\alpha}{\cos\alpha} = 4$

$2\,tg\alpha + 3 = 4$

$2\,tg\alpha = 1 \quad /:2$

$tg\alpha = \frac{1}{2}$

Zad. 32 <4 pkt.>



$A=(5;-\frac{5}{3})$

$k: y=\frac{4}{3}x$

(1) $B \wedge D \in k$

$S=(x_S;y_S)=?$

$P_{ABCD}=?$

(1) $B \wedge D \wedge S \in k: \quad S=(x_S; \frac{4}{3}x_S)$

(2) $k: \quad y=\frac{4}{3}x \quad / \cdot 3$

$k: \quad 3y=4x$

$k: \quad 4x-3y=0 \quad \wedge \quad A=(5;-\frac{5}{3})$

$$|AS| = d_{(A;k)} = \frac{|4\cdot 5+3\cdot\frac{5}{3}|}{\sqrt{4^2+3^2}} = \frac{25}{\sqrt{25}} = 5$$

$$|AS| = \frac{a\sqrt{2}}{2} = 5 \quad \Rightarrow \quad a = 5\sqrt{2}$$

$$P_{ABCD}=(5\sqrt{2})^2=50$$

(3) $|AS|^2=(x_S-5)^2+\left(\frac{4}{3}x_S+\frac{5}{3}\right)^2 \wedge x_S=x$

$$5^2 = x^2-10x+25+\frac{16}{9}x^2+\frac{40}{9}x+\frac{25}{9}$$

$$\frac{25}{9}x^2-\frac{50}{9}x+\frac{25}{9}=0 \quad / \cdot \frac{9}{25}$$

$$x^2-2x+1=0$$

$$(x-1)^2=0$$

$$x=x_S=1 \quad \Rightarrow \quad y_S=\frac{4}{3}\cdot 1=\frac{4}{3}$$

$$S=(1;1\tfrac{1}{3})$$

Zad. 33 <4 pkt>

$a_n = a_1 q^{n-1}$

$a_n > 0$

(1) $6a_1 - 5a_2 + a_3 = 0$

D: $q \in \langle 2\sqrt{2}; 3\sqrt{2} \rangle$

$q = ?$

$2\sqrt{2} \approx 2{,}83$

$3\sqrt{2} \approx 4{,}24$

(1) $6a_1 - 5a_1 q + a_1 q^2 = 0$

$a_1 \cdot (6 - 5q + q^2) = 0$

$a_1 = 0$ sprzeczne $\notin (0; \infty)$ $\vee$ $q^2 - 5q + 6 = 0$

$(q-3)\cdot(q-2) = 0$

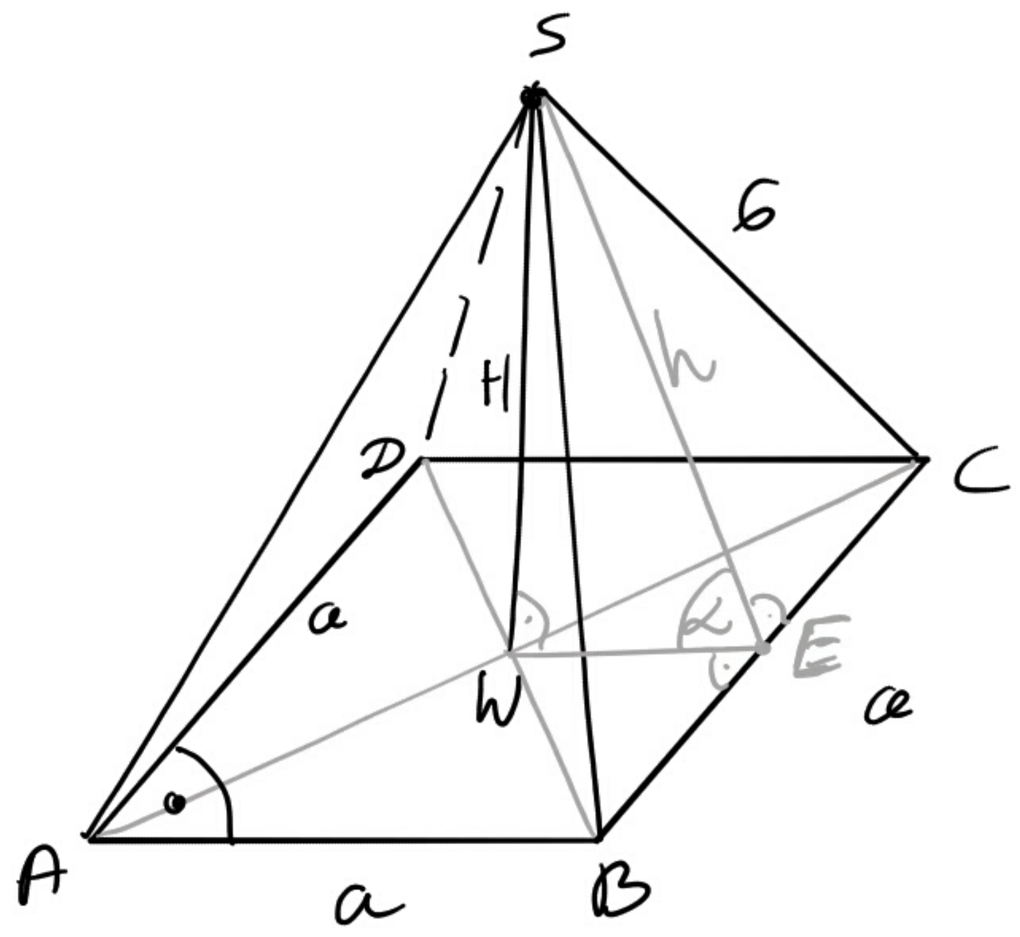
$q = 3 \in D$ $\vee$ $q = 2 \notin D$

Odp: $q = 3$

Zad. 34 <5 pkt.>

$V = ?$

$h = |SE|$
$H = |SW|$
$tg\alpha = \sqrt{7}$



(1) $tg\alpha = \sqrt{7}$

$\frac{|WS|}{|WE|} = \sqrt{7}$

$\frac{H}{\frac{1}{2}a} = \sqrt{7} \Rightarrow \underline{H = \frac{\sqrt{7}}{2}a} \Rightarrow H^2 = \frac{7}{4}a^2$

(2) ΔWCS:

$|SW|^2 + |WC|^2 = |CS|^2$

$H^2 + \left(\frac{a\sqrt{2}}{2}\right)^2 = 6^2$

$H^2 = 36 - \frac{a^2}{2}$

$\underline{H^2 = \frac{72 - a^2}{2}}$ $\longrightarrow$

(3)

$\frac{7a^2}{4} = \frac{72 - a^2}{2} \;/ \cdot 4$

$7a^2 = 144 - 2a^2$

$9a^2 = 144 \;/ : 9$

$a^2 = 16 \;/\sqrt{}\ \wedge\ a > 0$

$\underline{a = 4}$ $\Longleftarrow$

$H = \frac{\sqrt{7}}{2} \cdot 4$

$\underline{H = 2\sqrt{7}}$

(4) $V = \frac{1}{3} P_p \cdot H = \frac{1}{3} a^2 \cdot H = \frac{1}{3} \cdot 4^2 \cdot 2\sqrt{7}$

$V = \frac{32\sqrt{7}}{3}$